8. RYZHOV O.S. and SAVENKOV I.V., Asymptotic theory of the wave packet in a boundary layer on a plate, PMM, 51, 5, 1987.
9. RYZHOV O.S. and SAVENKOV I.V., The three-dimensional disturbances introduced by a harmonic oscillator into the boundary layer on a plate, Zh. vych. Mat. i mat. Fiz., 28, 4, 1988.
10. GAPONOV S.A. and MASLOV A.A., Development of disturbances in compressible flows, Nauka, Novosibirsk, 1980.
11. ZHIGULYEV V.N. and TUMIN A.M., The occurence of turbulence: dynamic theory of disturbances and the development of instability in boundary layers, Nauka, Novosibirsk, 1987.
12. RYZHOV O.S. and ZHUK V.I., Internal waves in the boundary layer with the selfinduced pressure, J. Mech., 19, 3, 1980.
13. RYZHOV O.S. and ZHUK V.I., On the stability of a compressible boundary layer against three-dimensional perturbations with selfinduced pressure gradient, in current problems in computational fluid dynamics, Mir, Moscow, 1986.
14. RYZHOV O.S., On the non-stationary three-dimensional boundary layer interacting freely with an external flow, PMM, 44, 6, 1980.
15. TERENT'YEV E.D., Calculation of the pressure in the linear problem of a vibrator in a supersonic boundary layer, PMM, 43, 6, 1979.
16. CRIMINALE W.O. and KOVASZNAY L.S.G., The growth of localized disturbances in a laminar boundary layer, J. Fluid Mech., 14, Pt. 1, 1962.
17. GASTER M., Propagation of linear wave packet in laminar boundary layers, AIAA Journal, 19, 4, 1981.
18. KOSINOV A.D. and MASLOV A.A., Development of artificially caused disturbances in a supersonic boundary layer, Akad. Nauk SSSR, MZhG, 5, 1984.

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ON NON-STATIONARY MOTIONS OF LOCAL INHOMOGENEITIES IN A PSEUDOFLUIDIZED LAYER*

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The growth (collapse) of a moving local inhomogeneity in the concentration of particles in a pseudofluidized layer is investigated. The inhomogenity is modelled using a spherical packet of particles /1-3/. The mass of the packet and the distribution of the particles throughout its volume remain constant. The density of the solid phase is assumed to be large compared with the density of the fluidizing fluid while the interaction between the phases is assumed to be linear with respect to the velocity of the relative motion of the phases. The simplest model, where there is no exchange between the particles in the packet and the particles in the layer, is analysed.

As a result of the approximate solution of the problem on the motion of a packet of variable radius, a system of equations is obtained which relates the change in the size of the packet with the velocity of its motion in the layer and the rate of circulation of the disperse phase in it. The velocity and pressure fields inside and outside the packet are found and the stationary states of the system are determined. It is shown that, unlike the case of bubbles where there is always a unique stationary state /4, 5/, the number of stationary states of the packet can vary depending on the physical parameters of the pseudofluidized system.

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Perturbations, due to the motion of inhomogeneities in the concentration of the solid phase in the pseudofluidized layer are, as is well-known /6-8/ an important factor which determines the efficiency of mass and heat exchange processes. The quasistationary motions of packets have been considered previously /1-3/.

1. Formulation of the problem. Let us introduce a non-inertial, spherical system of coordinates associated with the centre of a packet of radius $a(t)$ and a polar axis which coincides with the direction of the gravitational acceleration vector. (Fig.1,a). The velocity of motion of the packet in the laboratory reference system $x_1O_1y_1$ is equal to $\mathbf{U}_d(t)$. On the basis of the above-mentioned assumptions and within the framework of the model of reciprocally permeating ideal fluids, we write the system of equations of motion and continuity for the liquid and solid phases outside and inside the packet as:

$$\begin{aligned}
& r > a(t), \quad \mathbf{v}(\mathbf{r}, t) - \mathbf{w}(\mathbf{r}, t) = -k(\varepsilon)\nabla p_f(\mathbf{r}, t), \quad \nabla \mathbf{v}(\mathbf{r}, t) = 0 \\
& d_s\rho\,[\partial/\partial t + \mathbf{w}(\mathbf{r}, t)\nabla]\,\mathbf{w}(\mathbf{r}, t) = -\nabla[p_f(\mathbf{r}, t) + p_s(\mathbf{r}, t)] \pm d_s\rho U_d^{\cdot}(t)\,\mathbf{g}/g + d_s\rho\mathbf{g} \\
& \qquad \nabla\mathbf{w}(\mathbf{r}, t) = 0, \quad \varepsilon + \rho = 1 \\
& \qquad r < a(t), \quad \mathbf{v}'(\mathbf{r}, t) - \mathbf{w}'(\mathbf{r}, t) = -k'[\varepsilon'(t)]\nabla p_f'(\mathbf{r}, t) \\
& \qquad \partial\varepsilon'(t)/\partial t + \varepsilon'(t)\nabla\mathbf{v}'(\mathbf{r}, t) = 0 \\
& \qquad d_s\rho'(t)\,[\partial/\partial t + \mathbf{w}'(\mathbf{r}, t)\nabla]\,\mathbf{w}'(\mathbf{r}, t) = \\
& \qquad\quad -\nabla[p_f'(\mathbf{r}, t) + p_s'(\mathbf{r}, t)] \pm d_s\rho'(t)\,U_d^{\cdot}(t)\,\mathbf{g}/g + d_s\rho'(t)\,\mathbf{g} \\
& \qquad \partial\rho'(t)/\partial t + \rho'(t)\nabla\mathbf{w}'(\mathbf{r}, t) = 0 \\
& \qquad \varepsilon'(t) + \rho'(t) = 1
\end{aligned} \tag{1.1}$$

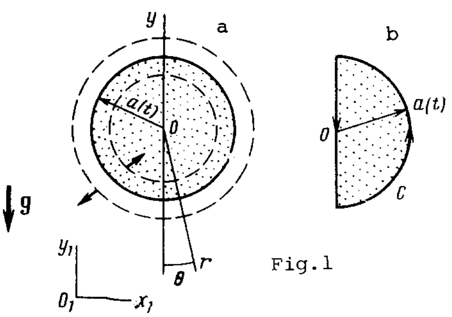

Fig.1

Here, $\mathbf{v}, \mathbf{w}$; p_f, p_s; ε, ρ are the locally averaged velocities, pressures and bulk concentrations of the fluidizing agent (density d_f) and the dispersed particles (density d_s, $d_f/d_s \ll 1$) respectively, $k(\varepsilon)$ is the permeability of the pseudofluidized layer /9/ and $\mathbf{g}$ is the acceleration due to gravity. The parameters of the two phase flow within the packet are indicated by a prime. In the third and seventh equations of (1.1), the terms which are proportional to $\pm U_d^{\cdot}(t)$ describe the inertial force which acts on a unit volume of the disperse phase in the selected non-inertial coordinate system. Here and everywhere subsequently where nothing is stated to the contrary, the upper sign corresponds to a floating packet $(\rho'(t) > \rho)$, and the lower sign to a sinking packet $(\rho'(t) > \rho)$, $U_d(t) = |\mathbf{U}_d(t)|$ where there is an alternation of the signs.

In the non-inertial coordinate system being considered, we shall write the boundary conditions on the surface of the packet in the form /10-12/

$$\begin{aligned}
& r = a(t), \quad w_r - D = w_r' - D = 0, \quad \varepsilon(v_r - D) = \varepsilon'(v_r' - D) \\
& p_f = p_f', \quad p_s' - p_s = d_s[\rho(w_r^2 - D^2) - \rho'(w_r'^2 - D^2)] = 0
\end{aligned} \tag{1.2}$$

The first two of these are the conditions that there should be no flow of the disperse phase through the surface of the packet, the third expresses the conservation of the flow of the liquid phase on the discontinuity and the last two are the conditions for the balance of the normal stresses in the liquid and solid phases respectively. The first and second conditions have been taken into account in writing down the last condition.

In relationships (1.2), D is the velocity of motion of the discontinuity in the concentration of the particles (the velocity of motion of the surface of the packet) in a coordinate system which moves together with the centre of the packet.

The equation of the surface of discontinuity has the form $F(r, \theta, \varphi, t) \equiv r - a(t) = 0$. Hence $\mathbf{D} = -\mathbf{i}_r(\partial F/\partial t)/|\nabla F| = \mathbf{i}_r a^{\cdot}$ ($\mathbf{i}_r$ is the unit vector in the radial direction), whence $D = |\mathbf{D}| = a^{\cdot}$.

We also require that the flows of the phases remote from the packet should be homogeneous and that their velocities should be bounded over the whole flow domain.

In the special case when there are no particles in the packet $(\rho'(t) = 0)$, the formulation adopted corresponds to the problem on the growth (collapse) of a bubble in a pseudofluidized layer considered in /4, 5/. As experiments show /7, 8/, together with the motion of bubbles, the motions of formations such as packets (clusters, clots) of particles under known conditions are an important element in the picture of interphase interaction and can have a considerable effect on the mass- and heat-exchange characteristics in an inhomogeneous pseudofluidized system. The formulation described in paragraph 1 enables one to investigate the characteristics of the simplest non-stationary motions of packets of particles as a function of a number of parameters of the system to which we add, in comparison with bubbles, the total mass of the particles making up the packets and the intensity of their internal circulation. In particular, the analysis carried out below shows that the above-mentioned parameters determine the nature and number of the stationary states of the packet. It is well-known /4/ that, in the case of a bubble, there is a unique stationary state with a radius a_* and that, if $a(t) > a_*$, the bubble grows while, if $a(t) < a_*$, the bubble collapses.

We also note that the adopted formulation of the problem on the non-stationary motion of packets can be extended to pseudofluidized systems of coarse particles when the interphase interactions are non-linear /5/.

2. The velocity fields and the pressure distributions of the phase outside and within the packet. Let us consider the case when the motion of the disperse phase outside of the packet is potential: $\mathbf{w}(\mathbf{r}, t) = \nabla \varphi_s(\mathbf{r}, t)$. Then, as follows from the fourth equation of (1.1), the potential $\varphi_s(\mathbf{r}, t)$ is a harmonic function

$$\Delta\varphi_s(\mathbf{r}, t) = 0 \tag{2.1}$$

which satisfies the conditions

$$r = a, \quad \partial\varphi_s(\mathbf{r}, t)/\partial r = w_r(\mathbf{r}, t) = a^{\cdot} \tag{2.2}$$
$$r \to \infty, \quad \varphi_s(\mathbf{r}, t) \to \varphi_s^{\circ}(\mathbf{r}, t)$$

where $\varphi_s^{\circ}(\mathbf{r}, t)$ is the potential of a flow which is homogeneous at infinity.

The solution of problem (2.1), (2.2) has the form

$$\varphi_s(\mathbf{r}, t) = \pm U_d(t)\left(1 + \frac{a^3(t)}{2r^3}\right) r\cos\theta - \frac{a^2(t)\,a^{\cdot}}{r} \tag{2.3}$$

From the condition of the continuity of the solid phase within the packet, we obtain $\nabla \mathbf{w}' = (\partial\varepsilon'(t)/\partial t)/(1 - \varepsilon'(t)) \equiv \zeta(t)$. Let us now construct a particular solution of the non-stationary problem (1.1), (1.2) under the assumption that the flow field of the solid phase within the packet is a superpositioning of the solution of the corresponding quasistationary solution and the flows corresponding to a homogeneous expansion (contraction) of the packet with a velocity $a^{\cdot}$, that is, we represent the velocity vector of the particles within the packet in the form of a sum

$$\mathbf{w}'(\mathbf{r}, t) = \mathbf{w}_1'(\mathbf{r}, t) + \mathbf{w}_2'(\mathbf{r}, t) \tag{2.4}$$

When this is done $\nabla \mathbf{w}_1' = 0$ and the velocity field $\mathbf{w}_1'$ corresponds to a Hill spherical vortex with a current radius $a(t)$ while the vector $\mathbf{w}_2'$ only has a radial component and $\nabla \mathbf{w}_2' = \zeta(t)$.

The flow function, corresponding to the velocity field $\mathbf{w}_1'$ has the form

$$\psi_s'(\mathbf{r}, t) = \pm \frac{3U_d'(t)}{4a^2(t)} r^2(r^2 - a^2(t))\sin^2\theta \tag{2.5}$$

where the sign determines the direction of the velocity of the solid phase within the packet (the upper sign corresponds to internal circulation directed at the centre of the packet in the opposite direction to the vector $\mathbf{g}$ while the lower sign corresponds to circulation in the opposite direction) and the parameter $U_d'(t)$ characterizes the intensity of the motion of the particles in the packet. So, the circulation of the velocity of the solid phase along the closed "liquid" contour C depicted in Fig.1,b is equal to $\mp 5U_d'(t)\,a(t)$ and the correspondence of the signs is the same as that in formula (2.5).

From the equation for the continuity of the solid phase within the packet, we obtain

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 w_{2r}') = \zeta(t)$$

that is,

$$w_{2r}' = \frac{r\zeta(t)}{3} + \frac{A}{r^2}, \quad A = \text{const} \tag{2.6}$$

It follows that $A = 0$ from the condition of the boundedness of the velocity $\mathbf{w}'$. Let us now verify that the condition that there should be no flow of the solid phase on the boundary of the packet is satisfied. It is obvious that $w_r'|_{r=a} = w_{2r}'|_{r=a}$ $(w_{1r}'|_{r=a} = 0$: $\psi_s'|_{r=a} = 0)$. From the condition that the mass of the packet should be conserved, we have $a_0^3\rho_0' = a^3(t)\,\rho'(t)$ where a_0 and ρ_0' are certain initial values of the radius of the packet and the bulk concentration of the solid particles in it. Hence,

$$\rho'(t) = 1 - \varepsilon'(t) = \frac{\rho_0' a_0^3}{a^3(t)}, \quad \frac{\partial\varepsilon'(t)}{\partial t} = -\frac{\partial\rho'(t)}{\partial t} = \frac{3\rho_0' a_0^3 a^{\cdot}(t)}{a^4(t)}$$
$$\zeta(t) = 3a^{\cdot}(t)/a(t)$$

Consequently, $w_r'|_{r=a} = w_{2r}'|_{r=a} = a(t)\,\zeta(t)/3 = a^{\cdot}(t)$.

The adopted model for the expansion (contraction) of the packet is only consistent with the equation of motion for the solid phase within it when the condition

$$U_d^{\cdot}\, a + U_d' a^{\cdot} = (U_d' a)^{\cdot} = 0 \tag{2.7}$$

is satisfied.

Condition (2.7) has the meaning of conservation of the scale $\Gamma = U_d' a$ of the circulation of the velocity of the disperse phase in the packet. The circulation of the velocity of the particles along an arbitrary fluid contour traced out within the packet is simultaneously conserved as a consequence of the validity of Thomson's theorem in the adopted model of the phases as ideal fluids.

Let us now determine the pressure fields of the liquid and solid phases. As a result of the application of the operation of divergence to the equations of motion of the fluidizing agent and taking account of the continuity equations outside of and within the packet, we obtain the equations

$$\begin{aligned} &r > a(t), \quad \Delta p_f(\mathbf{r}, t) = 0 \\ &r < a(t), \quad \Delta p_f'(\mathbf{r}, t) = \chi(t) = \frac{\partial \varepsilon'(t)/\partial t}{\varepsilon'(t)\,\rho'(t)\,k'[\varepsilon'(t)]} \end{aligned} \tag{2.8}$$

for the pressures of the liquid phase.

We shall represent the solution of the first equation of (2.8) in the form of a series in Legendre polynomials

$$p_f(\mathbf{r}, t) = \frac{v_0}{k(\varepsilon)} r\cos\theta + \sum_{n=1}^{\infty} B_n(t)\, P_{n-1}(\cos\theta)\, r^{-n} a^{n+1} + p_{f\infty} \tag{2.9}$$

In writing out series (2.9) account has been taken of the fact that the suspended layer is homogeneous remote from the packet and that $(\partial p_f/\partial y)|_{r\to\infty} = d_s \rho g r\cos\theta = (v_0/k(\varepsilon))\, r\cos\theta$, where v_0 is the rate of pseudofluidization, $p_{f\infty}$ is the pressure of the fluid phase in the equatorial plane of the packet at a large distance away from it and $y = -r\cos\theta$.

We shall seek the solution of the second equation of (2.8) in the form

$$p_f'(\mathbf{r}, t) = p_{f1}'(\mathbf{r}, t) + p_{f2}'(\mathbf{r}, t) \tag{2.10}$$

Here, $p_{f1}'(\mathbf{r}, t)$ is a harmonic function which satisfies the condition of the continuity of the normal stresses in the fluid phase on the surface of the packet (the fourth condition of (1.2)):

$$\begin{aligned} &\Delta p_{f1}'(\mathbf{r}, t) = 0, \quad p_{f1}'(\mathbf{r}, t)|_{r=a} = p_f'(\mathbf{r}, t)|_{r=a} = \\ &\quad \sum_{m=0}^{\infty} C_m(t)\, a(t)\, P_m(\cos\theta') \\ &C_0(t) = p_{f\infty}/a(t) + B_1(t), \quad C_1(t) = B_2(t) + v_0/k(\varepsilon),\ C_2(t) = \\ &\quad B_3(t), \ldots \end{aligned} \tag{2.11}$$

(θ' is the angular coordinate of a variable point on the sphere $r = a(t)$).

The second term in (2.10) is the solution of Poisson's equation with a null boundary condition

$$\Delta p_{f2}'(\mathbf{r}, t) = \chi(t), \quad p_{f2}'(\mathbf{r}, t)|_{r=a} = 0 \tag{2.12}$$

The functions $B_n(t)$ are to be determined from the boundary conditions.

By expanding the solution of the Laplace equation in spherical functions and integrating over the surface of the packet allowing for the boundary condition in (2.11), we find

$$p_{f1}'(\mathbf{r}, t) = \sum_{m=0}^{\infty} a(t)\, C_m(t) \left(\frac{r}{a(t)}\right)^m P_m(\cos\theta)$$

Next, by using Green's function of the first boundary value problem for Poisson's equation in the spherical domain $0 \leqslant r \leqslant a(t)$, we obtain the solution of problem (2.12) in the form

$$p_{f2}'(\mathbf{r}, t) = \chi(t)\,(r^2 - a^2(t))/6$$

Hence, the pressure distribution of the fluid phase within the packet has the form

$$p_f'(\mathbf{r}, t) = \sum_{m=0}^{\infty} a(t)\, C_m(t) \left(\frac{r}{a(t)}\right)^m P_m(\cos\theta) + \chi(t)\frac{r^2 - a^2(t)}{6} \tag{2.13}$$

By differentiating relationships (2.6) and (2.10) and using the condition that the flow of the fluid phase through the surface of the packet is continuous, we finally get

$$p_f(\mathbf{r}, t) = \frac{v_0}{k(\varepsilon)} r\cos\theta + B_1(t)\frac{a^2}{r} + B_2(t)\frac{a^3}{r^2}\cos\theta + p_{f\infty} \tag{2.14}$$

$$p_f'(\mathbf{r}, t) = a(t)B_1(t) + \left[B_2(t) + \frac{v_0}{k(\varepsilon)}\right] r\cos\theta +$$

$$\chi(t)\frac{r^2 - a^2(t)}{6} + p_{f\infty}$$

$$B_1(t) = -\frac{a^{\cdot}(t)}{\varepsilon k(\varepsilon)}, \quad B_2(t) = \frac{v_0}{k(\varepsilon)}\frac{\varepsilon k(\varepsilon) - \varepsilon'(t)k'[\varepsilon'(t)]}{2\varepsilon k(\varepsilon) + \varepsilon'(t)k'[\varepsilon'(t)]}$$

In the stationary case $B_1 = 0$ and, in the expression for B_2 the quantity $\varepsilon' = \text{const}$, which is identical with the result obtained previously /1/.

The flow fields of the fluid phase outside and within the packet are determined from the corresponding equations of motion with the aid of the pressure distribution (2.14) which have been found and the flow fields of the disperse phase (2.3)-(2.6) which have been constructed.

3. The link between the velocity of the motion of the packet, the law governing its growth (collapse) and the intensity of the internal circulation of the solid phase. On account of the assumption that the packet has a spherical shape, the condition for the continuity of the pressure of the solid phase on its surface can only be approximately satisfied in the neighbourhood of the frontal points of the packet ($\theta = 0$ in the case of a sinking packet and $\theta = \pi$ in the case of a floating packet). We recall that a similar situation also arises in problems concerned with the motion of a gas bubble in an ideal fluid /13/ and in a pseudofluidized layer /14, 15/.

The pressure distributions of the disperse phase over the surface of the packet, which correspond to the flow field of the solid particles which have been constructed and the pressure field of the fluid phase, have the form

$$p_s(\mathbf{r}, t)|_{r=a} = \frac{d_s\rho}{2}\left(U_d^2 - a^{\cdot 2} - \frac{9}{4}U_d^2\sin^2\theta\right) - \tag{3.1}$$

$$d_s\rho\left(\pm\frac{3}{2}aU_d^{\cdot}\cos\theta \pm \frac{3}{2}a^{\cdot}U_d\cos\theta - 2a^{\cdot 2} - aa^{\cdot\cdot}\right) +$$

$$p_{f\infty} + p_{s\infty} - p_f(\mathbf{r}, t)|_{r=a} + d_s\rho\left(\pm\frac{U_d^{\cdot}}{g} + 1\right)ga\cos\theta$$

$$p_s'(\mathbf{r}, t)|_{r=a} = p_{s0}'(t) + d_s\rho'(t)\left[\left(\pm\frac{U_d^{\cdot}}{g} + 1\right)ga\cos\theta - \right.$$

$$\left.\frac{1}{2}\left(a^{\cdot 2} + \frac{9}{4}U_d'^2\sin^2\theta\right) - \frac{aa^{\cdot\cdot} - a^{\cdot 2}}{2}\right] - p_f'(\mathbf{r}, t)|_{r=a}$$

Here $p_{s\infty}$ is the pressure of the solid phase at infinity and the function $p_{s0}'(t)$ is determined from the condition $p_s'(r, t)|_{\rho'=0} = 0$ which leads to the relationship

$$p_{s0}'(t) = -(\varepsilon k)^{-1}/a(t)a^{\cdot}(t) + p_{f\infty}$$

When $\theta \approx \pi$ and $\theta \approx 0$, we shall use the notation $\sin^2\theta = \delta$. Then, $\cos\theta = \mp 1 \pm \delta/2 + O(\delta^2)$. By only retaining the zeroth- and first-order terms in the expansion in δ of the jump in the pressure of the disperse phase on the boundary of the packet

$$[p_s] = p_s'(\mathbf{r}, t)|_{r=a} - p_s(\mathbf{r}, t)|_{r=a}$$

and allowing for the condition that the scale of the circulation of the solid phase within the packet is conserved (2.7), we obtain the following system of ordinary differential equations which relate the change in the size of the packet $a(t)$, its velocity of motion in the layer $U_d(t)$ and the characteristic velocity of circulation of the disperse particles in it $U_d'(t)$:

$$U_d'^{\cdot}a + U_d'a^{\cdot} = 0 \tag{3.2}$$

$$\left(1 - \frac{\rho'}{2\rho}\right)aa^{\cdot\cdot} - \frac{aa^{\cdot}}{\varepsilon k\rho d_s} - \frac{3}{2}a^{\cdot 2} - \frac{9}{4}\left[\frac{\rho'}{\rho}U_d'^2 - U_d^2\right] -$$

$$\frac{U_d^2}{2} - \frac{p_{s\infty}}{d_s\rho} = 0$$

$$\pm\left(\frac{\rho'}{\rho} - 1\right)\left(\pm\frac{U_d^{\cdot}}{g} + 1\right)ga + \frac{3}{2}(aU_d^{\cdot} + a^{\cdot}U_d) -$$

$$\frac{9}{4}\left[\frac{\rho'}{\rho}U_d'^2 - U_d^2\right] = 0$$

Passing to the limit in the last two equations of (3.2) as $p'(t) \to 0$, we get (the upper sign is required)

$$\frac{3}{2}a^{\cdot}U_d+\frac{1}{2}aU_d^{\cdot}+\frac{9}{4}U_d^2=ga$$

$$-aa^{\cdot\cdot}-\frac{3}{2}a^{\cdot 2}-\frac{p_{s\infty}}{d_s\rho}+\frac{7}{4}U_d^2=\frac{aa^{\cdot}}{\varepsilon k\rho d_s}$$

which, in the linear interphase interaction approximation, is identical with the system of equations of motion and the rate of growth of a bubble considered in /5/.

The system of Eqs.(3.2) has the stationary solution $\rho'=\rho_*'$, $a=a_*$, $U_d=U_{d*}$, $U_d'=U_{d*}'$ and

$$U_{d*}'a_*=\Gamma=\text{const},\quad -\frac{9}{4}\left(\frac{\rho_*'}{\rho}U_{d*}'^2-U_{d*}^2\right)=\frac{U_{d*}^2}{2}+\frac{p_{s\infty}}{d_s\rho} \tag{3.3}$$

$$\pm\left(\frac{\rho_*'}{\rho}-1\right)ga_*+\frac{U_{d*}^2}{2}+\frac{p_{s\infty}}{d_s\rho}=0$$

By eliminating the scale of the internal circulation velocity of the disperse phase U_{d*}' and the velocity of motion of the packet from the second equation of (3.3), we obtain an equation of the sixth degree for determining the stationary size of the packet a_* ($M={}^4/_3\pi a_*^3\rho_*'d_s$ is the mass of the packet).

$$a_*^6\mp\frac{9}{7}\frac{p_{s\infty}}{d_s\rho g}a_*^5-\frac{3M}{4\pi\rho d_s}a_*^3\mp\frac{27}{56}\frac{M\Gamma^2}{\pi\rho d_s g}=0 \tag{3.4}$$

The corresponding stationary velocities are:

$$U_{d*}'=\frac{\Gamma}{a_*},\quad U_{d*}=\frac{2}{\sqrt{7}}\left(\frac{p_{s\infty}}{d_s\rho}\right)^{1/2}\left(1+\frac{27}{16}\frac{M\Gamma^2}{\pi p_{s\infty}a_*^5}\right)^{1/2} \tag{3.5}$$

In the limiting case when $M=0$ (a bubble), it follows from relationships (3.4) and (3.5) that

$$a_*=\frac{9}{7}\frac{p_{s\infty}}{d_s\rho g},\quad U_{d*}=\frac{2}{\sqrt{7}}\left(\frac{p_{s\infty}}{d_s\rho}\right)^{1/2}$$

which corresponds to the results in /4, 5/.

Hence, the stationary size of the packet depends both on the parameters of the pseudo-fluidized system as well as on the total mass of the particles making up the packet and the magnitude of the circulation of the solid phase within it. The stationary radius of the packet must satisfy the conditions

$$a_{*1}<a_{*2}<a_*,\ \rho_*'<\rho \tag{3.6}$$

$$a_{*1}\leqslant a_*<a_{*2},\ \rho_*'>\rho$$

Here a_{*1} is the minimum possible radius of a packet of fixed mass: $\rho_*'=1$, $a_{*1}^3=3M/(4\pi d_s)$; a_{*2} is the radius at which a given mass of particles is formed into a packet with a density equal to the density of the homogeneous part of the layer $\rho_*'=\rho$, $a_{*2}^3=3M/(4\pi\rho d_s)$.

Let us now consider the question of the number of stationary states of a packet of particles in a number of special cases when Eq.(3.4) is simplified.

The case $p_{s\infty}=0$, $\rho\neq 0$. This condition corresponds to the assumption that there are no reciprocal collisions between the solid particles in the domain of homogeneous pseudofluidization and it is a weaker condition than the analogous conditions which were adopted, for example, in /10, 16/. In the case under consideration, the number of stationary sizes of the packet is determined by the magnitude of the unique dimensionless parameter $\varkappa={}^7/_6Mg/(\pi d_s\Gamma^2)$ and the concentration of the solid phase in the domain of the homogeneous layer. The corresponding diagrams are presented in Fig.2,a in the case of floating packets $(\rho_*'<\rho)$ and in Fig.2,b in the case of sinking packets $(\rho_*'>\rho)$. As the analysis of Eq.(3.4) shows, there is a unique stationary state in the case of a floating packet of any mass M and scale of internal circulation Γ. Let us now agree to depict the different combinations of the characteristic parameters in the form of the density points $\rho,\varkappa$. Then, when $\rho_*'<\rho$ at each point of the band $\Pi_\varkappa=\{\varkappa>0,\ 0<\rho<1\}$, there is a unique stationary size of the packet which satisfies the first condition of (3.6), $a_*^3=(a_{1*}^3/(2\rho))(1+\sqrt{1+4\rho/\varkappa})$.

The diagram changes qualitatively in the case of sinking packets. In this case, when $0<\rho<0.5$, two stationary states of the packet are possible which satisfy the second condition of (3.6). The stationary radii of the packet are expressed by the relationships

$$a_{*\max}^3=\frac{a_{1*}^3}{2\rho}\left(1+\sqrt{1-\frac{4\rho}{\varkappa}}\right),\quad a_{*\min}^3=\frac{a_{1*}^3}{2\rho}\left(1-\sqrt{1-\frac{4\rho}{\varkappa}}\right)$$

The corresponding domain in the half band $\{\varkappa>0,\ 0<\rho<0.5\}$ lies between the hyperbola

$\varkappa_1(\rho) = 1/(1-\rho)$ and the straight line $\varkappa_2(\rho) = 4\rho$ (Fig.2,b). This domain is lower-bounded by the domain of values of the parameters in which a sinking packet does not have a stationary size and is upper-bounded by a domain in which the stationary solution is unique and $a_* = a_{*\max} > a_{1*}$ $(a_{*\min} < a_{1*})$.

In the case of a dense homogeneous layer $(0.5 < \rho < 1)$ the half band $\{\varkappa > 0,\ 0.5 < \rho < 1\}$ is decomposed into two domains. The points located below the curve $\varkappa_1(\rho)$ correspond to the absence of stationary sizes in the case of a packet with the parameters M and Γ. The domain above $\varkappa_1(\rho)$ depicts pseudofluidized systems in which a sinking packet can be found in a unique stationary state $a_* = a_{*\max}$.

Hence, the parameters M and Γ only have an effect on the number of stationary states in the case of sinking packets, that is, when $\rho_*' > \rho$. Packets which are less dense with respect to the surrounding layer, which are similar to bubbles /4, 5/ have a unique stationary size. This conclusion also remains true in the cases considered below.

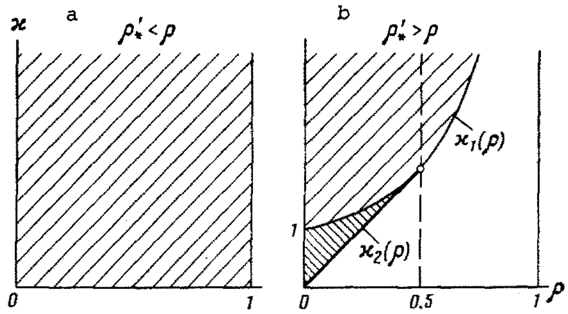

Fig.2

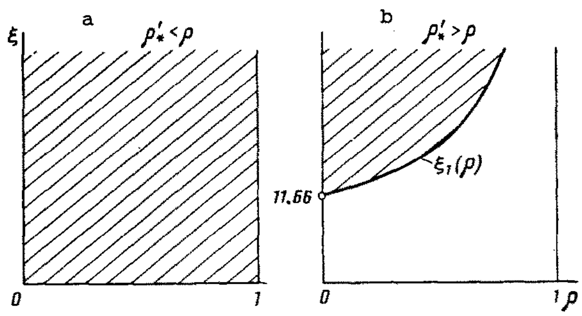

Fig.3

The case $\Gamma = 0,\ \rho \neq 0$. This condition denotes that there is no circulation of the solid phase within the packet. In this case the number of stationary states of a sinking packet is determined both by the concentration ρ of the disperse particles in the homogeneous layer and the value of the dimensionless parameter $\xi = (3M/(16\pi d_s))/(5p_{s\infty}/(14 d_s\ g))^3$ which characterizes the total mass of the particles making up the packet. The corresponding diagrams in the ρ, ξ-plane are shown in Fig.3: a) $\rho_*' < \rho$, b) $\rho_*' > \rho$. In the last case, the band $\Pi_\xi = \{\xi > 0,\ 0 < \rho < 1\}$ consists of two domains with a different number of stationary states of the packet. Actually, when

$$\xi_1(\rho) < \xi < \infty, \quad \xi_1(\rho) = \frac{5832}{500}\frac{1}{(1-\rho)^3}$$

there is a unique stationary state in the case of a sinking packet while, when $0 < \xi < \xi_1(\rho)$, the packet cannot be found in a stationary state for any value of the mass M of the particles constituting the packet. The unique stationary radius of the packet in cases a) and b) is determined by the unique positive root of the corresponding cubic equations

$$a_*^3 \mp \frac{9}{7}\frac{p_{s\infty}}{d_s \rho g} a_*^2 - \frac{3M}{4\pi\rho d_s} = 0$$

or, in the dimensionless form,

$$\rho \tilde{a}_*^3 \mp \frac{18}{5}\tilde{a}_*^2 - 4\xi = 0$$

Here,

$$\bar{a}_* = a_*/L,\ L = 5p_{s\infty}/(14d_s g)$$
$$M \neq 0,\ \Gamma \neq 0,\ p_{s\infty} \neq 0,\ \rho \neq 0$$

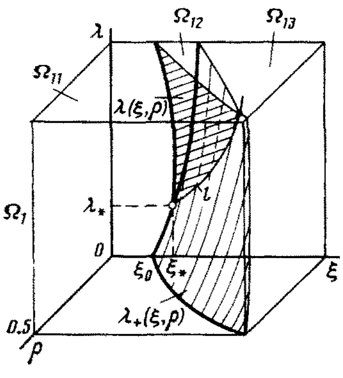

Fig.4

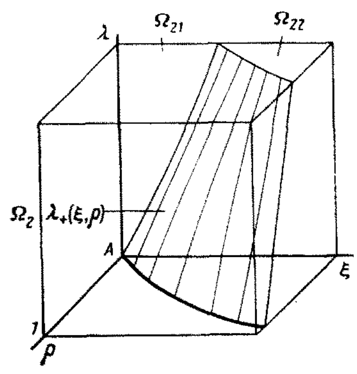

Fig.5

In the general case, Eq.(3.4) which determines the stationary dimensions of the packet, can be written in dimensionless form as follows:

$$\rho\bar{a}_*^6 \mp \frac{18}{5}\bar{a}_*^5 - 4\xi\bar{a}_*^3 \mp \frac{\xi\lambda}{5} = 0, \quad \bar{a}_* = \frac{a_*}{L} \tag{3.7}$$

Here, $\lambda = 90\Gamma^2/(7gL^3)$ is a dimensionless parameter which characterizes the intensity of the circulation of the disperse phase in the packet.

In the case of floating packets $(\rho_*' < \rho)$, Eq.(3.7) has a unique positive root, at each point of the spatial domain $\Omega = \{\xi > 0, \lambda > 0, 0 < \rho < 1\}$, which determines the stationary radius of a packet with a mass M and a circulation Γ. In the case of sinking packets $(\rho_*' > \rho)$ the nature of the multiplicity of stationary states is similar to that considered above in the case when $p_{s\infty} = 0,\ \rho \neq 0$. In fact, in the domain $\Omega_1 = \{\xi > 0,\ \lambda > 0,\ 0 < \rho < 0.5\}$ a packet cannot have a stationary size (domain Ω_{11}), has two stationary sizes (the domain Ω_{12}) or a single stationary size (the domain Ω_{13}), depending on the value of the mass and the scale of internal circulation of the disperse phase (see Fig.4). The surface $\lambda = \lambda(\xi, \rho)$ and $\lambda = \lambda_+(\xi, \rho)$, which subdivide the domain Ω_1 in the the above-mentioned three subdomains, are described by the relationships

$$\rho^3\lambda = \begin{cases} \lambda_1(\xi, \rho), & 0 < \tau \leqslant 2 \\ \lambda_2(\xi, \rho), & \tau \geqslant 2;\ \tau = \xi\rho^2 \end{cases}$$
$$\lambda_1(\xi, \rho) = 9\tau^{-1}\{2\tau - 1 + 4\cos{}^1/_3 \arccos(\tau - 1) \times$$
$$[1 - \cos{}^1/_3 \arccos(\tau - 1)]\}^2 - 16\tau$$
$$\lambda_2(\xi, \rho) = \tau^{-1}[6\tau - 3 + 3(2 - \zeta_+ - \zeta_-)(\zeta_+ + \zeta_-))]^2 - 16\tau$$
$$\xi_\pm = [\tau - 1 \pm ((\tau - 1)^2 - 1)^{1/2}]^{1/3}$$
$$\lambda_+(\xi, \rho) = 80(1 - \rho)\xi - 288\cdot 2^{-2/3}\xi^{2/3}$$

At the same time in the case of the points of that part of the surface $\lambda = \lambda(\xi, \rho)$ which lies below the curve $l = \lambda(\xi, \rho) \cap \lambda_+(\xi, \rho)$, a sinking packet with the corresponding values of the mass and circulation parameters ξ and λ does not have a stationary size while, on the remaining part of this surface, the stationary size is unique. When the curve l intersects the surface $\lambda = \lambda_+(\xi, \rho)$ from below, the number of stationary states of the packet changes from one to two. The coordinates ξ_* and λ_* of the points of the curve l in the concentration function ρ are given below:

ρ	0.01	0.1	0.2	0.3	0.4	0.45	0.5
$\lambda_*\cdot 10^{-3}$	1.85	3.57	8.80	31.1	259	$1.9\cdot 10^3$	∞
$\xi_*\cdot 10^{-3}$	0.057	0.106	0.250	0.844	6.75	493	∞
ξ_0	12.0	16.0	22.8	34.0	54.0	70.1	93.3

Extension of the surface $\lambda = \lambda_+(\xi, \rho)$ into the domain $\Omega_2 = \{\xi > 0,\ \lambda > 0,\ 0.5 \leqslant \rho < 1\}$ which adds Ω_1 to Ω, divides Ω_2 into two parts with different numbers of stationary states (Fig.5). At the points of the subdomain Ω_{21}, the values of the parameters of the system correspond to there being no stationary size in the case of a sinking packet. At the points of the subdomain Ω_{22} and on the corresponding part of the surface $\lambda_+(\xi, \rho)$ the values of the parameters correspond to the existence of a unique stationary size for a packet which satisfies the first condition of (3.6). We note that, in the case under consideration, $\rho_*' < \rho$ in the plane $\lambda = 0$, which bounds the domain Ω from below and the diagram given above in Fig.3, b: $\lambda_+(\xi, \rho) \cap \{\lambda = 0\} = \xi_1(\rho)$ is obtained.

4. Motion of a cluster of particles in the disperse medium. The model of the non-stationary motions of a packet in a pseudofluidized layer, proposed in paragraph 1, can be used to investigate the analogous non-stationary motions of a spherical cluster (cloud) of particles in a pure disperse medium. The corresponding equations of motion and continuity and the boundary conditions are obtained from relationships (1.1) and (1.2) in which it is necessary to put $\rho = 0$, $\rho_{s\infty} = 0$. These equations have the form

$$r > a(t),\ p_f(\mathbf{r}, t) = \mathrm{const} = p_{f\infty},\ \nabla\mathbf{v}(\mathbf{r}, t) = 0,\ \rho = 0 \tag{4.1}$$

$$r < a(t),\ \mathbf{v}'(\mathbf{r}, t) - \mathbf{w}'(\mathbf{r}, t) = -k'[\varepsilon'(t)]\nabla p_f'(\mathbf{r}, t)$$

$$\partial\varepsilon'(t)/\partial t + \varepsilon'(t)\nabla\mathbf{v}'(\mathbf{r}, t) = 0$$

$$d_s\rho'(t)[\partial/\partial t + \mathbf{w}'(\mathbf{r}, t)\nabla]\mathbf{w}'(\mathbf{r}, t) = -\nabla[p_f'(\mathbf{r}, t) + p_s'(\mathbf{r}, t)] - d_s\rho'(t)U_d^{\cdot}(t)\mathbf{g}/g + d_s\rho'(t)\mathbf{g}$$

$$\partial\rho'(t)/\partial t + \rho'(t)\nabla\mathbf{w}'(\mathbf{r}, t) = 0,\ \varepsilon'(t) + \rho'(t) = 1$$

$$r = a(t),\ w_r' = a^{\cdot},\ v_r - a^{\cdot} = \varepsilon'(v_r' - a^{\cdot}),\ p_f' = p_{f\infty},\ p_s' = 0 \tag{4.2}$$

In the case being considered the flow field of the disperse phase within the packet is described, as previously, by expressions (2.4)-(2.6). By passing to the limit as $\rho \to 0$ in the second equation of (2.14), we find the pressure distribution of the fluid phase within the packet in the form

$$p_f'(\mathbf{r}, t) = {}^1/_6\chi(t)(r^2 - a^2(t)) + p_{f\infty} \tag{4.3}$$

It follows from this and from the equation of motion of the fluid phase in the domain $r < a(t)$ that the angular components of the velocities of the particles and the gas are identical everywhere within the packet. We recall that, in the quasistationary approximation considered in /7/, there was no relative motion of the phases in the cluster: $\mathbf{v}'(\mathbf{r}, t) = \mathbf{w}'(\mathbf{r}, t)$, $r < a(t)$. In the case of the non-stationary motions of a cluster of a kind involving expansion (contraction) there is interphase slipping in the radial direction caused by an inflow of the fluid phase into the cluster (by its outflow from the cluster) when its radius changes.

If the flow of the fluid outside of the cluster is potential, that is, $\mathbf{v}(\mathbf{r}, t) = \nabla\varphi_f(\mathbf{r}, t)$, then, using expression (4.3) for the potential $\varphi_f(\mathbf{r}, t)$, we obtain the following problem (it is assumed that the fluid phase remote from the cluster is immobile in the xO_1y system):

$$\Delta\varphi_f(\mathbf{r}, t) = 0,\ \partial\varphi_f/\partial r|_{r=a} = v_r|_{r=a} = a^{\cdot}(t) - \varepsilon'(t)k'[\varepsilon'(t)]\partial p_f'/\partial r|_{r=a} = a^{\cdot}(t) - {}^1/_3\varepsilon'(t)k'[\varepsilon'(t)]\cdot\chi(t)a(t) = 0 \tag{4.4}$$

The solution of (4.4) has the form

$$\varphi_f(\mathbf{r}, t) = -U_d(t)\left(1 + \frac{a^3(t)}{2r^3}\right)r\cos\theta$$

From the second relationship of (3.1) as $\rho \to 0$, we obtain the pressure distribution of the solid phase over the surface of the cluster $p_s'(\mathbf{r}, t) = d_s\rho'(t)[(-U_d^{\cdot}/g + 1)ga\cos\theta - {}^1/_2(a^{\cdot 2} + {}^9/_4U_d^{\prime 2}\sin^2\theta) - {}^1/_2(a^{\cdot\cdot}a - a^{\cdot 2})]$.

The system of differential equations relating $a(t)$, $U_d(t)$ and $U_d'(t)$ is written in the form

$$U_d'a = \mathrm{const} = \Gamma \tag{4.5}$$

$$-{}^1/_2a^{\cdot\cdot} + (-U_d^{\cdot}/g + 1)g = 0,\ (-U_d^{\cdot}/g + 1)ga + {}^9/_4U_d^{\prime 2} = 0$$

By eliminating U_d and U_d' from Eqs.(4.5), we arrive at the following law governing the growth, (contraction) of the cluster:

$$a^3a^{\cdot\cdot} = -\gamma^2,\ \gamma^2 = {}^9/_2\Gamma^2 \tag{4.6}$$

In the case of the general integral of Eq.(4.6) we obtain the relationship

$$a^2(t) = [(k_1t + k_2)^2 - \gamma^2]/k_1 \tag{4.7}$$

where k_1 and k_2 are arbitrary constants defined by the initial conditions for the motion of the cluster:

$$k_1 = (a_0^2 a_0^{\cdot 2} - \gamma^2)/a_0^2, \ k_2 = a_0 a_0^{\cdot}$$

and

$$a_0 = a(t)|_{t=0}, \ a_0^{\cdot} = (da(t)/dt)|_{t=0}$$

are the initial size and rate of change in the size of the cluster.

When there is no internal circulation of the solid phase in the cluster $(\Gamma = 0)$ we find from Eq.(4.7) that $a(t) = a_0^{\cdot} t + a_0$. It follows from the second equation of (4.5) that, in the case under consideration, the centre of the cluster moves as a free falling body: $U_d^{\cdot} = g$. The cluster has a unique stationary size $a_* = a_0$ subject to the condition $a_0^{\cdot} = 0$ and, when $a_0^{\cdot} > 0$ $(a_0^{\cdot} < 0)$, its radius increases linearly with time (decreases down to the minimum possible value of a_{*1}).

Circulatory motion of the disperse particles making up the cluster leads to an increase in its rate of falling in the gas. In this case, the second equality of (4.5) and Eq.(4.6) yield $U_d^{\cdot} = g - {}^1/_2 a^{\cdot\cdot} > g$. This result is an agreement with the experimental data /17-19/ on the settling of dilute suspensions under conditions of the so-called dynamic formation of clusters.

As can be seen from Eq.(4.6) when $\Gamma \neq 0$, internal circulation of the solid phase leads to the absence of a stationary size for the cluster. An analysis of the solution of (4.7) shows that, in the case of clusters, the intensity of circulation in which satisfies the condition $\gamma < a_0 a_0^{\cdot}$, the rate of change in the radius has the sign of $a_0^{\cdot}$ over the whole time of the motion, that is, the cluster contracts when $a_0^{\cdot} < 0$ and expands when $a_0^{\cdot} > 0$. Clusters with a high intensity of internal circulation $\gamma > a_0 a_0^{\cdot}$, when $a_0^{\cdot} < 0$ possess the same property. In the latter case, if the cluster is expanding at the initial instant of time $(a_0^{\cdot} > 0)$, then it will expand up to the instant $t = a_0^3 a_0^{\cdot}/(\gamma^2 - a_0^2 a_0^{\cdot 2})$, when $a^{\cdot}(t) = 0$, after which this expansion is replaced by a compression in accordance with the law (4.7).

In two special cases we obtain from (4.7):

1) $\gamma = a_0 a_0^{\cdot}$, the scale of the circulation of the solid phase in the cluster is specified by the initial condition:

$$a^2(t) = 2\gamma t + a_0^2$$

2) $a_0^{\cdot} = 0$, there are no pulsations at the initial instant of time:

$$a^2(t) = a_0^2 - \gamma^2 a_0^{-2} t^2$$

If the cluster contracts at $t = 0$ $(a_0^{\cdot} < 0)$ or $a_0^{\cdot} = 0$, it attains the minimum size a_{*1} $(\rho' = 1)$ after a time t_* which is defined by the relationship

$$t_* = -\{[a_0^2 a_{*1}^{\cdot 2} + \gamma^2(1 - a_{*1}^2/a_0^2)]^{1/2} + a_0 a_0^{\cdot}\} a_0^2 (a_0^2 a_0^{\cdot 2} - \gamma^2)^{-1}$$

from which, in the special cases considered above, it follows that:

1) $\gamma = 0, \ a_0^{\cdot} < 0, \ t_* = -(a_0 - a_{*1})/a_0^{\cdot}$

$a_0^{\cdot} = 0, \ t_* = a_0 \gamma^{-1} \sqrt{a_0^2 - a_{*1}^2}$

and $t_*(\Gamma = 0) = \infty$ which corresponds to a unique state of the cluster $a = a_0$.

2) $\gamma = a_0 a_0^{\cdot}, \ a_0^{\cdot} < 0, \ t_* = -(a_0^2 - a_{*1}^2)/(2a_0 a_0^{\cdot})$.

Since the state of a cluster with a radius $a = a_{*1}$ is non-stationary, it may be postulated that, at the instant of time $t = t_*$, the rate of contraction $(a^{\cdot}(t_*))$ instantaneously changes its sign to the opposite sign after which the cluster begins to expand in accordance with the quantitative relationship involving the parameters $a_{*1}, a^{\cdot}(t_*), \gamma$.

The investigation of the non-stationary motions of packets of particles which has been presented above must be augmented by an analysis of the stability of the corresponding stationary states. It has been shown in /4/ that the stationary size $a = a_*$ of bubbles in a pseudofluidized layer is unstable: the larger bubbles $(a > a_*)$ increase in size while the smaller bubbles $(a < a_*)$ decrease in size.

It should be emphasized that, in the model which has been adopted and does not take account of exchange of solid particles between the packet and the external layer, the surface of the packet constitutes a tangential discontinuity in the velocity of the disperse phase. It is known /20/ that such discontinuities are always unstable in single phase fluids. It would be of interest to investigate the effect of the interphase interactions on the nature of the stability of tangential discontinuities of the type under consideration in inhomogeneous pseudofluidized systems.

REFERENCES

1. BOBKOV N.N. and GUPALO YU.P., On a packet mechanism of motion in a boiling later, Izv. Akad. Nauk SSSR, Mekh Zhid. i Gaza, 5, 1983.

2. BOBKOV N.N. and GUPALO YU.P., On the motion of solid and fluid phases close to a local concentration inhomogeneity in a pseudofluidized layer, Teoret. Osnovy Khim. Tekhnologii, 18, 4, 1984.
3. BOBKOV N.N. and GALIYEVA L.M. and GUPALO YU.P., On the motion of the inhomogeneities in a developed boiling layer at low Reynolds numbers, Izv. Akad. Nauk SSSR, Mekh. Zhid. i Gaza, 4, 1984.
4. BUYEVICH YU.A., On the motion of bubbles in a pseudofluidized layer, Izv. Akad. Nauk SSSR, Mekh. Zhid. i Gaza, 3, 1975.
5. GUPALO YU.P., RYAZANTSEV YU.S. and SERGEYEV YU.A., Growth of a bubble in a suspended layer with non-linear interphase interaction, Izv. Akad. Nauk SSSR, Mekh. Zhid. i Gaza, 6, 1977.
6. MICKLEY H.S. and FAIRBANKS F., Mechanism of heat transfer to fluidized beds, AIChE Journal, 2, 3, 1955.
7. YERUSHALMI J., CANCURT N.T., GELDART D. and LISS B., Flow regimes in vertical gas-solid contact systems, AIChE Symp. Ser., 74, 176, 1978.
8. ROZENBAUM R.B. and TODES O.M., The Motion of Bodies in a Pseudofluidized Layer, Izd. Leningrad State University (LGU), Leningrad, 1980.
9. DAVIDSON I.F. and HARRISON D., The Pseudofluidization of Solid Particles /Russian translation/, Khimiya, Moscow, 1965.
10. KRAIKO A.N. and STERNIN L.E., On the theory of flows of a two velocity continuous medium with solid or fluid particles, Prikl. Matem. i Mekh., 29, 3, 1965.
11. BUYEVICH YU.A. and GUPALO YU.P., On surfaces of discontinuity in disperse systems, Prikl. Matem. i Mekh., 34, 4, 1970.
12. GOL'DSHTIK M.A., Transport Processes in a Granular Layer, Izd. Inst. Teplofiziki Sib. Otd. Akad. Nauk SSSR, Novosibirsk, 1984.
13. DAVIES R.M. and TAYLOR G., The mechanics of large bubbles rising through extended liquids and through liquids in tubes. Proc. Roy. Soc., A200, 1062, 1950.
14. DAVIDSON I.F. and HARRISON D., Pseudofluidization /Russian translation/, Khimiya, Moscow, 1974.
15. BORODULYA V.A. and GUPALO YU.P., Mathematical Models of Chemical Reactors with a bubbling layer, Nauka, i Tekhnika, Minsk, 1976.
16. JACKSON R., The mechanics of fluidized beds. Part II: the motion of fully developed bubbles, Trans. Instn. Chem. Engrs., 41, 1, 1963.
17. KOGLIN B., Settling rate of individual particles in suspension, Proc. Conf. Particle Size Analysis, Bradford, 1970; Soc. Analyt. Chem., London, 1972.
18. KOGLIN B., AL-TAWEEL A. and AHMAD N., Konfiguration und Sinkgeschwindigkeit von hydrodynamischen Komplexen Kugelformiger Partikeln, Chem. Ing. Techn., 48, 6, 1976.
19. ADACHI K., KIRIYAMA S. and YOSHIOKA N., The behaviour of a swarm of particles moving in a viscous fluid, Chem. Engng. Sci., 33, 1, 1978.
20. LANDAU L.D. and LIFSHITS E.M., Theoretical Physics, 6, Hydrodynamics, Nauka, Moscow, 1986.

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